

# Snow College Mathematics Contest

April 7, 2015

Senior Division: Grades 10-12

Form: **T**

Bubble in the single best choice for each question you choose to answer.

1. Chuck, Jackie, and Bruce are all celebrating their birthdays on the same day. Bruce's present age is seven years less than the sum of Chuck's and Jackie's present ages. In five years, Bruce will be twice as old as Jackie will be then. Three years ago, Jackie was one-third as old as Chuck was. What is the sum of all of their current ages?

- (A) 25
- (B) 31
- (C) 39
- (D) 48
- (E) 53

2. If  $x^2 + 1$  is a factor of  $6x^3 + 5x^2 + Px + Q$ , then what is  $P + Q$ ?

- (A) 10
- (B) 11
- (C) 12
- (D) 13
- (E) 14

3. There are 3 cards in a bag; one has X on both sides, one has O on both sides, and the third has X on one side and O on the other. If you draw a card at random and look at one side and see an X, what is the probability that the other side is also X?

- (A)  $\frac{1}{4}$
- (B)  $\frac{1}{3}$
- (C)  $\frac{1}{2}$
- (D)  $\frac{2}{5}$
- (E)  $\frac{2}{3}$

4. A rotation in the  $xy$ -plane through an angle  $\theta$  about the origin is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

What are the coordinates  $(x', y')$  of the image of the point  $(3, 2)$  after a rotation through  $-90^\circ$ ?

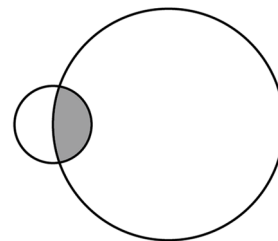
- (A)  $(3, 2)$
- (B)  $(-3, 2)$
- (C)  $(3, -2)$
- (D)  $(2, 3)$
- (E)  $(2, -3)$

5. Evaluate the product:  $\log_2(125) \cdot \log_5(16)$

- (A) 200
- (B) 20
- (C) 12
- (D) 10
- (E) 10.5

6. Two overlapping circles have radii 1 and 3. If the shaded area is  $\frac{\pi}{2}$ , then what is the total area of the figure?

- (A)  $10\pi$
- (B)  $\frac{19\pi}{2}$
- (C)  $8\pi$
- (D)  $\frac{7\pi}{2}$
- (E)  $9\pi$

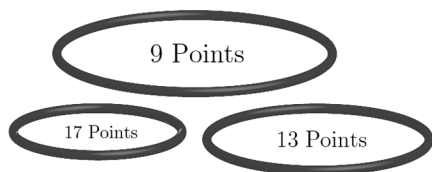


7. If you place one penny on the first square of an  $8 \times 8$  checkerboard, two pennies on the second square, four pennies on the third square, and keep doubling the number of pennies for each square, how many total pennies will be on the entire board?

- (A)  $2^{64}$
- (B)  $2^{63}$
- (C)  $2^{63} + 2^{62}$
- (D)  $2^{65}$
- (E)  $2^{64} - 1$

8. George and Lucas are playing a game where they toss balls into three hoops. The smallest hoop is worth 17 points, the middle-sized hoop is worth 13 points, and the largest hoop is worth 9 points. George lost with only 84 points. What is the minimum number of balls he must have thrown?

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8



9. Say we call a date mm/dd/yy *magical* if  $mm \times dd = yy$ . How many of the following dates can **never** be magical?

- January 31
- February 29
- March 31
- April 30

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

10. Consider the point on the unit circle with an angle  $\theta = \pi/4$  rad (ccw from the  $+x$ -axis). Where does the line through that point and tangent to the circle intersect the  $y$ -axis?

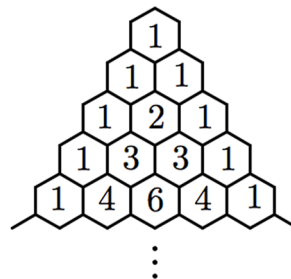
- (A)  $(0, 0)$
- (B)  $(0, \sqrt{2})$
- (C)  $(0, 1)$
- (D)  $(0, 2)$
- (E)  $(\sqrt{2}, 0)$

11. Express the continued fraction expression for  $x$  as a simple closed-form number.

$$x = 1 + \frac{6}{2 + \frac{6}{2 + \frac{6}{2 + \frac{6}{\ddots}}}}$$

- (A)  $\sqrt{7}$
- (B)  $\frac{1-\sqrt{2}}{2}$
- (C)  $\frac{1+\sqrt{2}}{2}$
- (D)  $2 + \frac{\sqrt{2}}{2}$
- (E)  $\frac{4}{\sqrt{2}}$

12. Here is the start of Pascal's triangle.



Here is another part that would show up further down. Fill in the numbered spots.

- (A) 1) 20 2) 1 3) 56
- (B) 1) 15 2) 21 3) 70
- (C) 1) 15 2) 35 3) 80
- (D) 1) 20 2) 21 3) 70
- (E) 1) 15 2) 35 3) 56



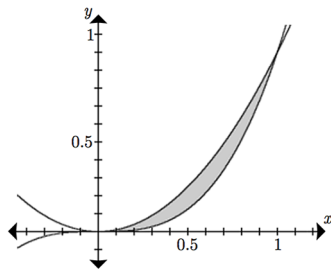
13. Assume the following statements are true.
- If it rains, we will bake a cake.
  - If it doesn't rain, we will play football.
  - If we play football, I will get muddy.
  - Mother will get angry if I get muddy.
  - I did not get muddy.

Which is a valid conclusion?

- (A) Mother is not angry.  
 (B) We baked a cake.  
 (C) We did not bake a cake.  
 (D) It did not rain.  
 (E) We played football.
14. When a flock of sheep are driven through Sanpete Sally's land near Ephraim, she charges a toll of 10¢ per riderless animal (i.e., the sheep and dogs) and 50¢ for each rider and horse pair. One day Sally counted a total of 4168 legs (including riders, horses, dogs, and sheep) and 1044 heads. How much money did Sally collect?
- (A) \$105.60  
 (B) \$123.80  
 (C) \$305.20  
 (D) \$518.40  
 (E) \$961.00

15. What is the area between  $y = x^2$  and  $y = x^3$  between  $x = 0$  and  $x = 1$ ?

- (A)  $\frac{1}{12}$   
 (B)  $\frac{\pi}{4}$   
 (C)  $\frac{1}{8}$   
 (D)  $\frac{2}{5}$   
 (E)  $\frac{4}{9}$



16. If  $2^4 + m^n = 2^5$ , then what is  $m + n$  if  $m, n$  are positive integers?
- (A) 6 or 17  
 (B) 4 or 6  
 (C) 5 or 8  
 (D) 16 or 17  
 (E) 4 or 8

17. I only heard my cat meow, hiss, and purr on one day out of the last 23 days. But I did hear him make at least one of these sounds each day. I heard him hiss on 6 days, purr on 12 days, and meow on 15 days. On 2 days, I heard him meow and hiss but not purr, and on 2 days I heard him purr and hiss but not meow. On how many days did I hear him meow and purr but not hiss?
- (A) 3  
 (B) 4  
 (C) 5  
 (D) 6  
 (E) 7

18. What is the range of the function?

$$f(x) = \sqrt{8 \sin^3 x + 17}$$

- (A)  $[0, \infty)$   
 (B)  $[0, 5]$   
 (C)  $[0, 23]$   
 (D)  $[3, 5]$   
 (E)  $[3, 23]$

19. What is the output of the following BASIC computer program?

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10 s = 0
20 for i = 1 to 3
30 for j = 1 to i
40 s = s+j
50 next j
60 next i
70 print s

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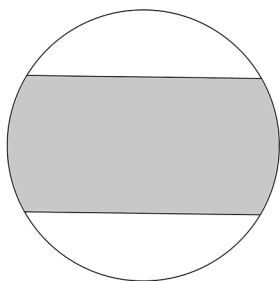
- (A) 0
- (B) 4
- (C) 10
- (D) 15
- (E) 20

20. Given same-size square matrices  $A$  and  $B$ , when is  $(A + B)^2 = A^2 + 2AB + B^2$ ?

- (A) always
- (B) never
- (C) only if  $A$  or  $B$  is the identity
- (D) only if  $A$  and  $B$  commute
- (E) only if  $A$  and  $B$  are upper triangular

21. Two congruent parallel chords are drawn a distance  $r$  apart in a circle of radius  $r$ . What is the area of the part of the circle that lies between the chords?

- (A)  $\left(\frac{\sqrt{3}}{3} + \frac{\pi}{2}\right)r^2$
- (B)  $\left(\frac{\sqrt{3}}{3} + \frac{\pi}{3}\right)r^2$
- (C)  $\left(\frac{\sqrt{3}}{2} + \frac{\pi}{2}\right)r^2$
- (D)  $\left(\frac{\sqrt{2}}{3} + \frac{\pi}{3}\right)r^2$
- (E)  $\left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right)r^2$



22. If  $f(x) = 2x + 3$  and  $g(x) = x^2 - 5$ , what is  $(f \circ g)(3)$ ?

- (A) 11
- (B) 15
- (C) 19
- (D) 23
- (E) 76

23. Which of these is a perfect square in every possible base  $b$  for that number? (e.g.,  $321_b$  is only a number when  $b > 3$ ).

- (A)  $36_b$
- (B)  $81_b$
- (C)  $144_b$
- (D)  $225_b$
- (E)  $3993_b$

24. If Bert had three more quarters, he would have twice as many quarters as Ernie. If Ernie had six more quarters, he would have three times as many quarters as Bert. If Bert and Ernie put their quarters together, how many quarters would they have?

- (A) 6
- (B) 8
- (C) 10
- (D) 12
- (E) 16

25. If  $\frac{a}{b+c+d} = \frac{4}{3}$  and  $\frac{a}{b+c} = \frac{3}{5}$ , then what is the value of  $\frac{d}{a}$ ?

- (A)  $\frac{7}{6}$
- (B)  $\frac{6}{7}$
- (C)  $-\frac{12}{11}$
- (D)  $-\frac{11}{12}$
- (E)  $\frac{15}{11}$

26. If  $3 \sin \theta + 4 \cos \theta = 5$ , then what is  $\tan \theta$ ?

- (A) 1
- (B)  $-1$
- (C)  $\frac{4}{3}$
- (D)  $\frac{3}{4}$
- (E) 0

27. If the line  $y = mx + k$  intersects the parabola  $y = ax^2 + bx + c$  at two points, what is the product of the two  $x$ -coordinates of these points in terms of  $a$ ,  $b$ ,  $c$ ,  $m$ , and  $k$ ?

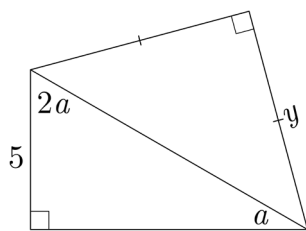
- (A)  $\frac{c+k}{a}$
- (B)  $\frac{ck}{a}$
- (C)  $\frac{c+k}{b}$
- (D)  $\frac{c-k}{b}$
- (E)  $\frac{c-k}{a}$

28. If  $x < 0$ , then what is  $\left| x - \sqrt{(x-1)^2} \right|$ ?

- (A) 1
- (B)  $-2x - 1$
- (C)  $1 + 2x$
- (D)  $1 - 2x$
- (E)  $2x - 1$

29. Solve for  $y$  in the figure.

- (A) 5
- (B)  $5\sqrt{2}$
- (C)  $5\sqrt{3}$
- (D) 10
- (E)  $2\sqrt{5}$



30. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x + y) = f(xy)$  for all real  $x$ ,  $y$ , and  $f(7) = 7$ . What is  $f(49)$ ?

- (A) 1
- (B) 7
- (C) 14
- (D) 21
- (E) 49

31. The base three representation of  $x$  is 121122111222. Find the leftmost digit of the base nine representation of  $x$ .

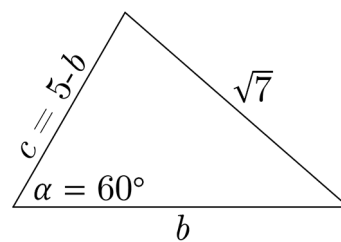
- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

32. A box contains 2 pennies, 4 nickels, and 6 dimes. Six coins are drawn at random without replacement, with each coin having equal probability of being chosen. What is the probability the value of the coins drawn is at least 50¢?

- (A)  $\frac{37}{924}$
- (B)  $\frac{91}{924}$
- (C)  $\frac{127}{924}$
- (D)  $\frac{132}{924}$
- (E)  $\frac{194}{924}$

33. A triangle has side  $a = \sqrt{7}$ , the opposite angle  $\alpha = 60^\circ$ , and the sum of the other two sides  $b + c = 5$ . Find the ratio of the longest to the shortest side of the triangle.

- (A) 1
- (B)  $\sqrt{2}$
- (C)  $\frac{3}{2}$
- (D)  $\frac{\sqrt{7}}{2}$
- (E) 2

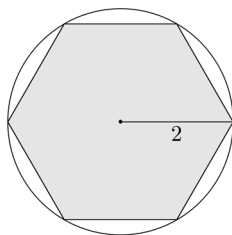


34. A six-sided die has faces labeled 1 through 6. It is weighted so that a three is three times as likely to be rolled as a one, a three and a six are equally likely, and a one, a two, a four, and a five are equally likely. What is the probability of rolling a three?

- (A)  $\frac{1}{6}$
- (B)  $\frac{1}{3}$
- (C)  $\frac{2}{3}$
- (D)  $\frac{3}{10}$
- (E)  $\frac{2}{5}$

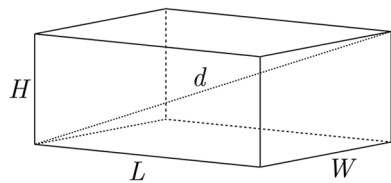
35. Find the area of a regular hexagon inscribed in a circle with radius 2.

- (A)  $6\sqrt{3}$
- (B) 12
- (C)  $3\sqrt{3}$
- (D)  $12\sqrt{3}$
- (E) 6



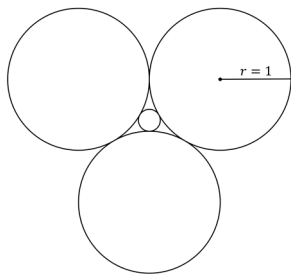
36. For a closed rectangular box  $L + W + H = 25$  cm. The surface area of the box is  $225$  cm<sup>2</sup>. Find the distance between opposite corners of the box; i.e., the largest distance between points on the box.

- (A) 12.5 cm
- (B) 15 cm
- (C) 20 cm
- (D) 25 cm
- (E) 27.5 cm



37. Consider three circles of radius 1, each tangent to the others. What is the radius of a fourth smaller circle in the middle which is tangent to each of them?

- (A)  $\frac{1}{4}$
- (B)  $\frac{2\sqrt{3}-3}{3}$
- (C)  $\frac{\sqrt{3}-1}{2}$
- (D)  $\frac{\sqrt{3}-1}{4}$
- (E)  $\frac{\sqrt{3}}{3}$



38. Suppose  $f$  is a linear function such that  $(f \circ f \circ f)(x) = 27x + 26$ . Find the  $y$ -intercept for the graph of  $f$ .

- (A) (0, 2)
- (B) (26, 0)
- (C) (0, 26)
- (D) (0, 13)
- (E) (0, 3)

39. A parallelogram with an area of 580 has vertices  $(0, 0)$ ,  $(5, 30)$ , and  $(20, a)$ . If  $0 < a < 100$ , then what is the value of  $a$ ?

- (A) 3
- (B) 4
- (C) 6
- (D) 7
- (E) 35

40. A college math class has  $N$  teaching assistants. It takes the assistants 5 hours to grade homework assignments. One day, another teaching assistant joins them in grading, and all the assignments take only 4 hours to grade. Assuming everyone did the same amount of work, compute the number of hours it would take 1 teaching assistant to grade all the homework assignments.

- (A) 20
- (B) 22
- (C) 24
- (D) 26
- (E) 28